Exercise 18

In Exercises 13 to 19, use set theoretic or vector notation or both to describe the points that lie in the given configurations.

The line passing through $(-5, 0, 4)$ and $(6, -3, 2)$

Solution

The equation for a line is

$$\mathbf{y}(t) = \mathbf{m}t + \mathbf{b},$$

where $\mathbf{m}$ is the direction vector, b is the position vector for a point on the line, and t is a parameter. Subtract the two given position vectors to get $\mathbf{m}$.

$$\begin{aligned}\mathbf{m} &= (-5, 0, 4) - (6, -3, 2) \\ &= (-11, 3, 2)\end{aligned}$$

So the line in question can be written as

$$\begin{aligned}\mathbf{y}(t) &= (-11, 3, 2)t + (6, -3, 2) \\ &= (-11t, 3t, 2t) + (6, -3, 2) \\ &= (-11t + 6, 3t - 3, 2t + 2)\end{aligned}$$

Since there's only one arbitrary constant t, the line is one-dimensional. The set of points on the line is described by

$$\{(-11t + 6, 3t - 3, 2t + 2),\ t \in \mathbb{R}\}.$$